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By

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A bi-level hierarchical method for shape and member sizing optimization of steel truss and frame structures

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ABSTRACT

This paper describes a new bi-level hierarchical method for optimizing the shape and member sizes of both determinate and indeterminate steel structures. The method utilizes a unique combination of algorithms that are organized hierarchically: the Fully Constrained Design (FCD) method for discrete sizing optimization is nested within SEQOPT, a gradient-based optimization method that operates on continuous shape variables. We benchmarked the method against several existing techniques using numerical examples and found that it compared favorably in terms of solution quality and computational efficiency. We also present a successful industry application of the method to demonstrate its practical benefits.

Keywords: structural optimization; size optimization; shape optimization; steel structures; frame; truss

1. Introduction

Engineers often are challenged to design truss and frame structures that are both economical and reliable. The design process involves specifying each of the following three aspects of the structure: (i) *topology*, which concerns the number and connectivity of members; (ii) *shape*, which pertains to the location of structural joints; and (iii) *sizing*, which involves defining member cross-sections [1]. The specification of each aspect of the structure typically corresponds to the three major stages of the engineering design process as defined by Pahl and Beitz [2]: conceptual, embodiment (design development) and detail. The topology of the structure is typically identified during conceptual design based on the functional requirements and architectural aesthetics, whereas the structure's shape and member sizing are determined during the design development and detailed design phases, respectively.

This paper presents a bi-level hierarchical method with a unique combination of algorithms to optimize the latter two aspects of the structure—shape and sizing—given a fixed topology. The objective of the optimization is to minimize the cost of the structure, while satisfying design performance requirements for safety and serviceability. In this case, the cost of the structure is estimated by multiplying the total steel weight by the price per unit. Steel weight is commonly used as a proxy for cost, provided that industry standard means and methods of construction are employed [3].

We treat shape variables as continuous in this investigation, meaning that any value can be assumed within the specified limits (e.g., allowing the depth of a truss to assume any value between, for example, 900 and 1,800 mm). Sizing variables, on the other hand, are discrete, meaning that only certain specified values can be assumed. This is consistent with industry practice where engineers commonly select structural member sizes from a set of standard steel profiles that are mass-produced in specific sizes (e.g., W14x132, W14x120, etc.) [4]. Typically, there is a cost premium and/or quantity requirement associated with using steel profiles that do not conform to these standard sizes [5].

Traditionally, shape and sizing optimization has been an iterative process that is performed manually by the engineer. The first step in the process is usually to define the initial shape and sizing configuration of the structure based on architectural requirements, engineering rules of thumb, and past experience. Next, an analytical model is created that includes an idealized representation of the structure's topology, shape, member sizes, and loading. The analytical model is used to calculate the structure's response to the defined loading (e.g., forces, deflections). These responses are then checked against the design requirements for safety and serviceability. Finally, the engineer reviews the results and may elect to modify either the shape of the structure or the sizes of constituent members.

The number of possible shape and sizing configurations for a given design problem is termed the *design space* [6]. The size of the design space is an exponential function of the number of design variables and the number of possible choices for each variable. For example, a problem with x variables and n discrete choices per variable has n^x possible configurations. The size of the design space for most problems encountered in industry is so large that it is impractical to explore all possible design alternatives [7]. Engineers using the manual methods described above customarily have time to evaluate only a few design alternatives [8]. Vast areas of the design space are, therefore, left unexplored even though they may contain better performing shape and sizing configurations [9].

Numerous formal optimization methods have been developed to improve upon traditional approaches by reducing design iteration time, thereby enabling the evaluation of a greater number of design alternatives that can lead to better quality solutions. The majority of formal methods surveyed consider only member sizing design variables [10]. The inherent coupling between size and shape variables, however, makes it more advantageous to consider both variable types simultaneously [11]. In Section 2, we survey existing shape and sizing optimization methods and discuss their respective strengths and limitations with regard to generality and efficiency.

The goal of the research presented in this paper was to develop a formal shape and sizing optimization method that (i) can generally be applied to problems with a mix of discrete and continuous variables and (ii) efficiently handles large variable sets that are typically encountered in industry. To achieve these objectives, the proposed method employs different optimization algorithms to operate on discrete sizing and continuous shape variables as discussed in Section 3. In Section 4, we benchmark this unique combination of algorithms against other leading approaches using two standard numerical examples. In Section 5, we present a successful industry implementation of the method on two large stadium roof trusses. Finally, in Section 6 we summarize the lessons learned and discuss the method's suitability for general industry application.

2. Shape and Sizing Optimization

Methods for shape and sizing optimization of trusses and frames generally can be categorized as either single-level or multilevel depending upon how the problem is decomposed.

2.1. Single-level Methods

Most structural optimization methods described in the literature are single-level approaches because a single optimization algorithm is used to operate on shape and sizing variables simultaneously. Although the analysis may be distributed, all design decisions are made by a single optimizer. Both deterministic and heuristic single-level methods are described and their respective limitations discussed below.

The deterministic methods that have received the most attention in the research community are stress-ratio (or fully stressed design), linear programming, nonlinear programming, and branch and bound methods [12]. The stress-ratio method seeks to proportion each member of a structure so that it is loaded to the maximum safe performance limit under the action of at least one of the applied load cases. This approach is applicable to stress and local buckling constrained structures. While the solution quality of the stress-ratio method has been shown to be sub-optimal and highly dependent on the start point of the optimization process, the method has been widely adopted in professional practice due to its simplicity in concept and implementation [13, 14]. The stress-ratio method may be considered to be part of the optimality criterion approach to structural design, and this more general concept has been the subject of considerable research for many years shield [15-17].

Linear programming was first applied to unconstrained shape and sizing optimization problems involving plane trusses subject to a single loading case [18]. A penalty function method was later developed and successfully applied to various constrained truss problems considering a cost objective function [19]. Sequential Linear Programming (SLP) methods have been applied to problems with multiple load cases and constraints on eigenfrequencies [20].

Linear programming approaches, however, result in severe approximation errors when applied to problems with nonlinear responses [21]. To reduce these errors, researchers developed an augmented Lagrange multiplier method that utilizes second order Taylor series expansions to express stress and displacement quantities in terms of shape and sizing variables [22, 23]. The efficiency of the method was later improved by using Taylor series expansions to approximate forces, rather than stresses and displacements [11].

The deterministic methods described above require the first derivative of the objective and constraint functions with respect to the design variables. Therefore, these methods are not readily applicable to problems where the objective and/or constraint functions are discontinuous or are not easily expressed in terms of the design variables [24]. These methods also assume continuity of the design variables. When a discrete solution is required, approximation techniques are used to generate discrete variable values from the continuous results. Researchers have shown that these approximations can result in solutions that are sub-optimal or even infeasible [21].

The classical branch and bound method was originally developed for linear problems [25], but has been subsequently adapted to nonlinear problems [26]. Compared to the techniques discussed above, this method is known to generate superior quality solutions at the expense of computational efficiency [27]. Various approaches for approximating structural responses have been tested to improve the computational efficiency of the method, but branch and bound remains more expensive than comparable mathematical programming approaches [27, 28].

In recent years, there has been significant research on the application of heuristic techniques to structural shape and sizing problems, including genetic algorithms [24, 29-31], simulated annealing [32], and evolutionary strategies [33]. These methods are capable of handling both discrete and continuous variables simultaneously, and there is no limitation on the continuity of the search space. Researchers have also demonstrated that heuristic techniques such as genetic and evolutionary algorithms can also be applied to conceptual structural design problems involving topology as well as shape and sizing variables [34, 35]. These methods allow for human input to guide the optimization process by manipulating algorithm parameters during the iterative process. A disadvantage of the heuristic methods is that they compare unfavorably to the deterministic methods discussed above in terms of computational efficiency [36].

2.2. Multilevel Methods

Multilevel formulations employ more than one optimization algorithm, with each algorithm operating on a specific set of variables. Relatively few multilevel methods have been applied to optimize the shape and sizing of truss and frame structures. Vanderplaats and Moses developed the alternating gradient method [37] that decomposes the problem into two separate, but dependent, design spaces: shape (joint coordinates) and sizing (member areas). While changing the joint coordinates, the member areas are treated as dependent variables and vice versa. The sizing variables are manipulated by the fully stressed design method, and the shape variables are manipulated by the constrained steepest descent method. This method was successfully applied to a variety of indeterminate truss structures. Pedersen independently used a similar approach [38].

Subsequently, Kripakaran [39] proposed a bi-level hierarchical formulation that utilizes different algorithms to operate on discrete size and shape variables, respectively. The member sizing method is a hybrid approach that combines the following two steps in sequence: (i) a deterministic “clustering” technique and (ii) a non-deterministic algorithm, similar to tabu search [40], that performs a local search to identify an optimal design. The sizing algorithm is nested within a deterministic algorithm that operates on the discrete shape variables (node coordinates). For each search step, the algorithm attempts a specified number of transitions that involve shifting the coordinates of one or multiple nodes based on a rectangular grid. The minimum weight configuration generated that satisfies the design constraints is then used as the initial design for the next iteration. Kripakaran’s approach was successfully applied to a series of two-dimensional truss problems and produced design configurations that were lighter than comparable single-level methods.

3. Proposed Bi-level Hierarchical Method

The proposed method for shape and sizing optimization can generally be applied to problems with a mix of discrete and continuous variables and efficiently handles large variable sets that are typically encountered in industry. Similar to Kirpakaran’s method, the proposed method decomposes the problem into two hierarchical levels: shape and sizing. It differs from these existing multi-level approaches by the algorithms used to operate on the shape and sizing variables, respectively (Figure 1). We call this method BiOPT.

The Fully Constrained Design (FCD) algorithm Flager [41, 42] was selected for sizing optimization because it effectively handles discrete variables and does not require continuity of the objective or constraint functions. FCD is more general than existing fully stressed design (as known as stress-ratio) methods in that it is capable of handling structures with global deflection constraints as well as stress and local buckling constraints. It has also been demonstrated to be scalable to large structures involving over one thousand design variables and multiple constraints.

The SEQOPT [43, 44] algorithm was chosen for shape optimization because it has been shown to be robust and efficient for problems with continuous variables, particularly when the objective and/or constraint functions are computationally expensive to evaluate. This is particularly relevant for indeterminate structures for which analytical gradients of shape design variables are not computable. SEQOPT is a gradient-based optimization method that uses surrogate models to accelerate the optimization process and a pattern search technique to avoid local minima. Each algorithm is described in more detail below.

We hypothesized that BiOPT's unique combination of algorithms would make it more computationally efficient than Kripakaran's approach, as well as other single-level heuristic techniques. In addition, the proposed method overcomes the limitations of the deterministic methods discussed in Section 2.1, namely: (i) the handling of discrete design variables and (ii) applicability to problems where the objective and/or constraint functions are discontinuous or are not expressed easily in terms of the design variables.

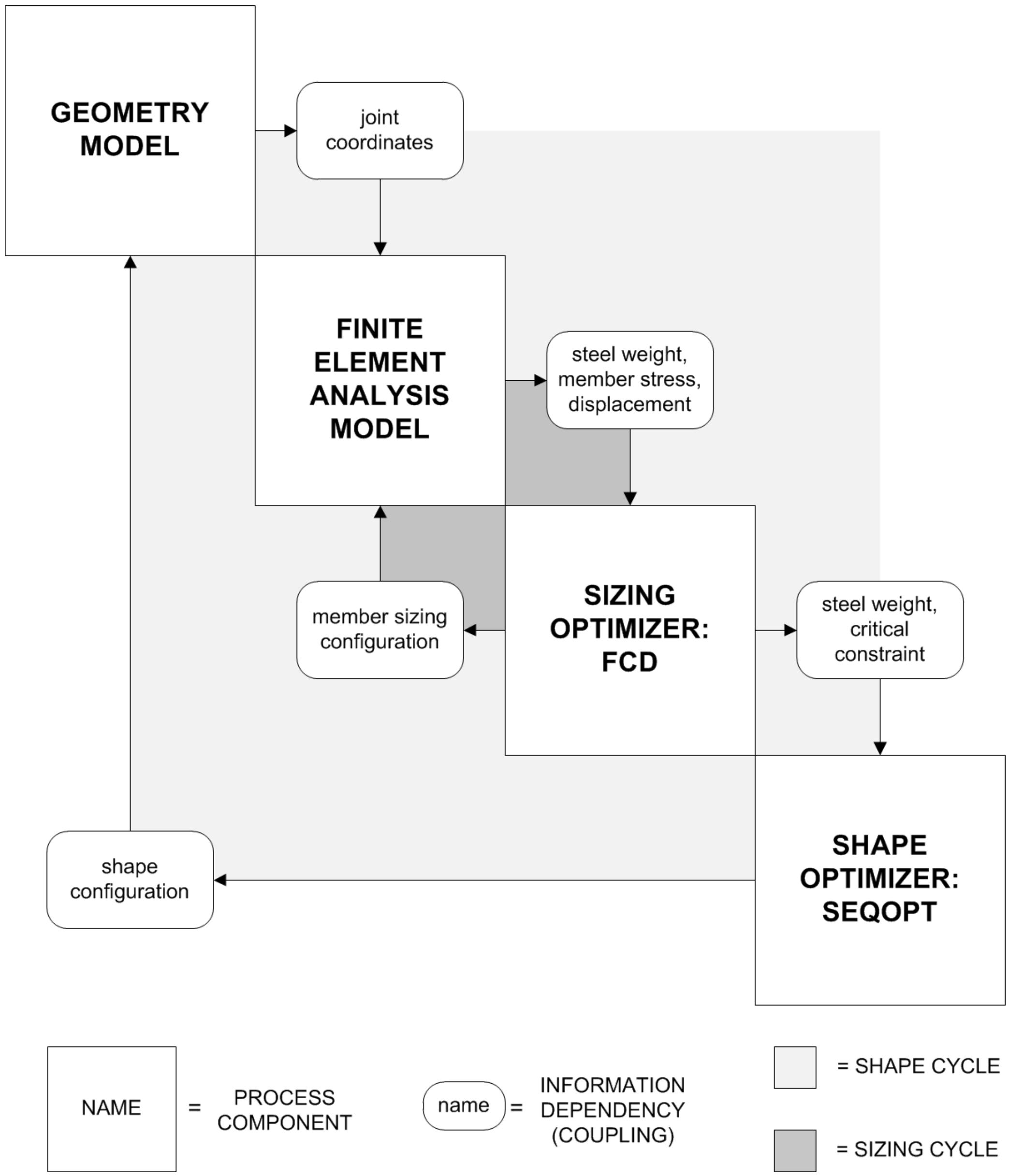

Figure 1: BiOPT architecture and data flow. Coupling above the diagonal represents sequential execution while coupling below the diagonal represents iteration.

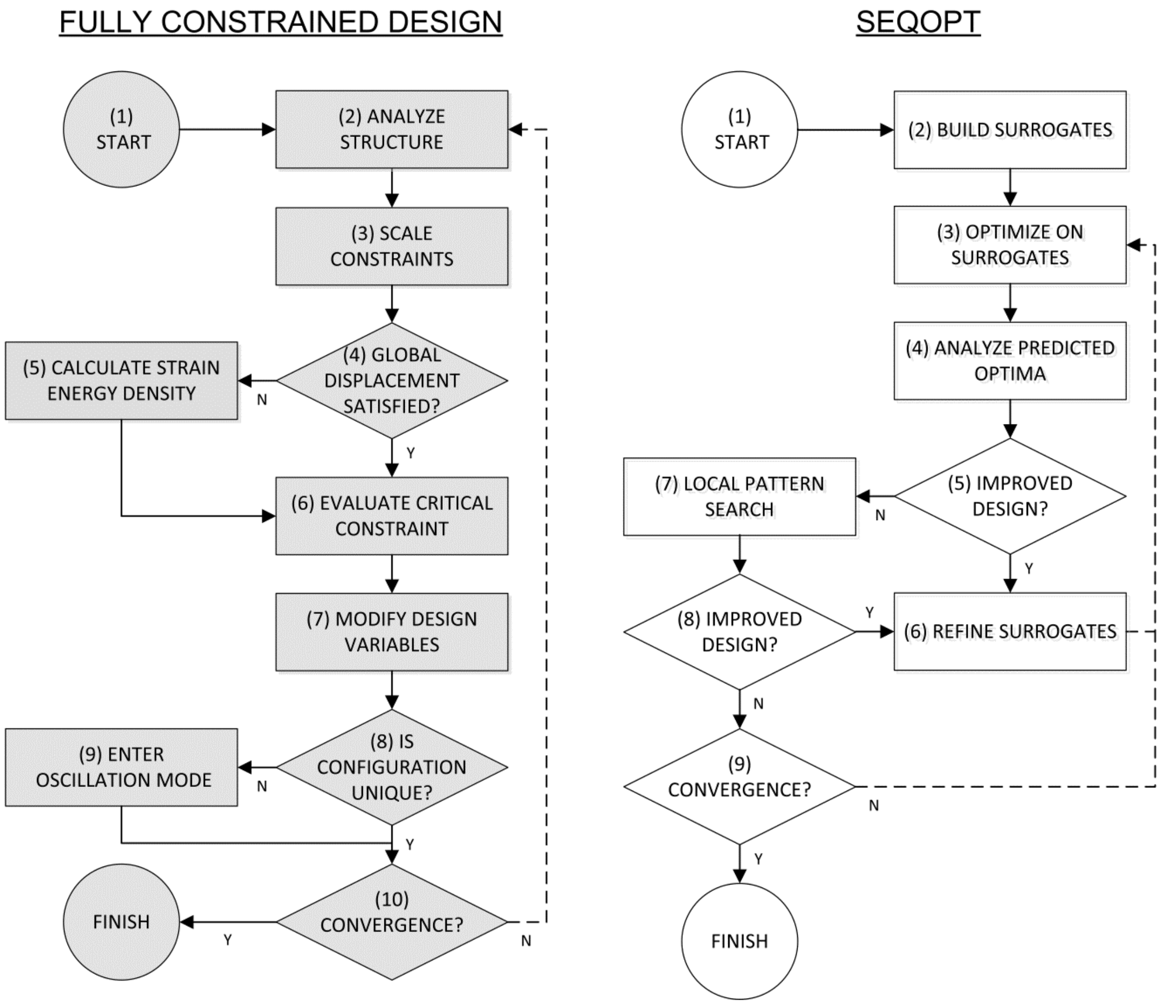

Figure 2: LEFT - Flowchart of the FCD algorithm (sizing optimization).

RIGHT - Flowchart of the SEQOPT optimization algorithm (shape optimization).

3.1. Sizing Optimization: Fully Constrained Design

For each configuration of shape variables generated, the Fully Constrained Design (FCD) method is employed to find the least-weight configuration of member sizes for that particular geometry. Based on the optimality criteria method, FCD features a novel approach to constraint handling and the generation of new designs that involves creating a one-to-one mapping between each member size design variable and a governing constraint. Based on the value of the governing constraint, the section size of each member variable is adjusted incrementally from an ordered list of choices. Figure 2 provides an overview of the FCD process. Each process step is summarized below. For a complete description, please refer to [41].

Step 1 - Start: The initial configuration of member sizes can either be chosen at random or be based on a previous design solution.

Step 2 - Analyze structure: Calculates the structure's response to the defined loading (e.g., member stress, member deflection, global deflection).

Step 3 - Scale constraints: Normalizes the structural responses by their respective allowable values for each sizing variable. A utilization factor of greater than unity indicates that the design variable configuration in question does no satisfy the problem constraints.

Step 4 - Is global displacement satisfied? In cases where the normalized global displacement constraint value is greater than unity (i.e., the constraint is violated) Step 5 becomes necessary.

Step 5 - Calculate strain energy density: A unit displacement is applied to the particular node in the structure where the maximum global displacement is observed and the maximum strain energy density for each member is calculated. Each value is then normalized by maximum member strain energy density calculated in the structure. Finally, the normalized strain energy density values are multiplied by the normalized global deflection scalar to calculate the constraint function for global displacement.

Step 6 - Evaluate critical constraint: Once all the constraint functions have been calculated, the values are compared to identify the critical constraint for each design variable.

Step 7 - Modify design variables: Each design variable has a corresponding set of possible values which describe the available list of discrete member section choices. These sets are ordered by section area from low to high. New designs are generated iteratively by adjusting the variable values up or down the corresponding ordered list based on the critical constraint value calculated for each design variable.

Step 8 - Is configuration unique? If the current design configuration is identical to a previous iteration of the optimization process, the optimizer enters 'oscillation mode'.

Step 9 - Enter oscillation mode: Individual design variables are perturbed based on their corresponding critical constraint values to avoid an infinite loop of repeated configurations.

Step 10 - Convergence? The optimization process concludes in one of four possible ways: (i) a Fully Constrained Design is achieved; (ii) a local / global optimum is reached; (iii) the number of iterations without improvement specified by the user is met; (iv) the maximum number of iterations specified by the user is met.

3.2. Shape Optimization: SEQOPT

The SEQOPT algorithm [43, 44] is employed to find the least weight configuration of shape variables for a fixed topology. The results of the sizing optimization process serve as input to SEQOPT, namely, the steel weight of the best sizing configuration found and the associated maximum constraint value. A key component of SEQOPT involves the construction of surrogate models that approximate the problem objective and constraint functions. These surrogate models can be evaluated very efficiently when compared to completing a FCD sizing optimization run. Global search mechanisms are implemented to avoid local minima. Figure 2 provides an overview of the SEQOPT process. Each process step is summarized below.

Step 1 - Start: An initial set of shape configurations are generated and evaluated using a given structural analysis method. Note that finite element analysis (FEA) is used for the example applications discussed in Sections 4 and 5. These initial shape configurations are not generated at random, but instead are chosen to canvas the design space efficiently using orthogonal arrays [45, 46].

Step 2 - Build Surrogates: After the initial set of design configurations have been evaluated, the resulting data are used to create surrogate models of the optimization objective and constraint functions. These surrogate models are interpolating Kriging models [47]. The surrogate models can be evaluated more efficiently than completing an FCD optimization run, and the objective and constraint functions are guaranteed to vary smoothly as a function of the input variables.

Step 3 - Optimize on Surrogates: A gradient-based Sequential Quadratic Programming (SQP) algorithm is run multiple times to locate optimum areas of the design space. The starting points for the algorithm are chosen at random.

Step 4 - Analyze Predicted Optima: SEQOPT keeps track of the unique local optima that were found by the SQP algorithm in Step 3. For each optimal point predicted by the surrogate model, a sizing optimization is run to determine the accuracy of these predictions.

Step 5, 8 - Improved design? An improved design configuration is found if the objective function value is superior to previous configurations and all the constraints have been satisfied.

Step 6 - Refine Surrogates: If the results predicted by the surrogate models do not match the actual results obtained from completing the FCD sizing optimization process, the actual results are used to refine and improve the surrogate models.

Step 7 - Local Pattern Search: The local pattern search is centered on the best configuration of sizing variables. The search proceeds by perturbing each design variable in turn until an improved configuration is found. If the local pattern search is unable to find an improved configuration, the current solution is guaranteed to be at least a local minimum within the tolerance of the pattern search.

Step 9 - Convergence? The optimization process concludes in one of two possible ways: (i) the number of iterations without improvement specified by the user is met; or (ii) the maximum number of iterations specified by the user is met.

4. Numerical Examples

Two standard shape and sizing problems from the literature are used to benchmark the performance of the proposed method: the 18-bar cantilever truss and the 77-bar truss bridge. The objective of each problem is to minimize steel weight while satisfying local and global constraints for strength and deflection, respectively. Both examples involve continuous shape and discrete sizing variables. We chose the 18-bar truss example because a variety of single-level and multi-level methods have been applied to it. We selected the 77-bar truss example because it possessed the largest design space of the examples found in the literature and, therefore, could be used to test the scalability of the method to a large variable set.

We compare BiOPT to other methods in terms of: (i) computational efficiency and (ii) solution quality. We measure computational efficiency in terms of the total number of finite element analyses required to arrive at the 'optimal' solution. We measure solution quality in terms of the steel weight of the 'optimal' solution in light of our objective to minimize this quantity.

Three different starting points for the member sizing optimization algorithm are used in the BiOPT optimization runs (Table 1). Only the best run is reported.

Starting Point	Description of design variable values
1	uniform: smallest section area
2	uniform: largest section area
3	uniform: median section area

Table 1: Initial sizing configuration used by BiOPT for the numerical examples.

4.1. Example 1: 18-bar truss

The 18-bar truss geometry is shown in Figure 3. A single load case is applied to the structure as described in Table 2. The material properties and constraints are described in Table 3. To make the application of the BiOPT method comparable to the optimization results reported in the literature, the same number of shape variables and the same formula to calculate stresses are used. Hence, eight shape variables are considered, corresponding to the X and Y coordinate values for the four nodes on the bottom chord of the truss (node numbers 3, 4, 7 and 9 in Figure 3). The cross-sectional areas of the candidate member sizes vary between 2 in^2 and 20 in^2 (1,250 to 12,500 mm^2) in steps of 0.25 in^2 (156.25 mm^2).

There is no displacement constraint specified for the problem. The allowable compressive (σ_i^c) and tensile (σ_i^t) stresses for each member i are calculated as follows:

$$\sigma_i^c = -K_i * E * A_i / L_i^2 \quad \text{, and} \quad \sigma_i^t = 20\ ksi \tag{1}$$

Where;

K_i = cross-sectional geometry constant of member i ($K_i = 4$ for this problem)

E = Young's modulus for the material

A_i = cross-sectional area of member i

L_i = length of member i

Two different member groupings are studied. For Case 1, the members are grouped into four design variables as follows: A1=A4=A12=A16; A2=A6=A10=A14=A18; A3=A7=A11=A15; A5=A9=A13=A17. For Case 2, all 18 members can assume a different section size (18 total sizing variables). For complete details on the problem, refer to [11].

In both cases, the best design configuration reported in the literature violated the specified constraints by a small margin. Therefore, two different optimization runs are conducted to provide an equivalent basis of comparison: A and B. For run A, the maximum allowable constraint value was unity, meaning that none of the constraints specified in the problem formulation are violated. For run B, a violation equivalent in magnitude to the best design reported in the literature is allowed.

The quality of the BiOPT solutions for Case 1 (Table 4) and Case 2 (Table 5) are superior to all existing solutions reported in the literature. An average weight savings of 3.5% is achieved over Kripakaran's method [39], which previously had produced the best results. Only Hansen and Vanderplaat's gradient-based approach with force approximations [11] is demonstrated to be more computationally efficient for Case 1. However, that approach produces solutions of inferior quality compared to the other approaches listed. BiOPT is more than twice as efficient as Kripakaran's method for Case 1 and over eight times more efficient for Case 2. Figure 4 shows the BiOPT convergence history for Case 2, as well as how the shape of the truss evolves during the optimization process.

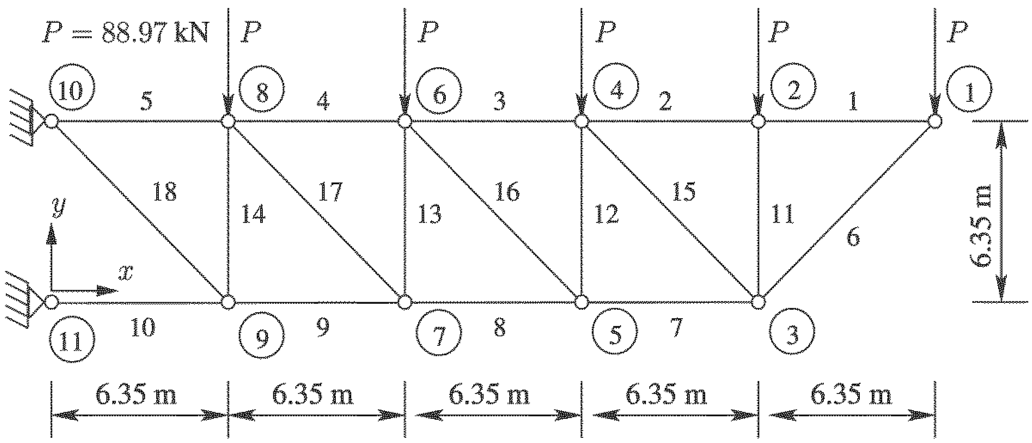

Figure 3: 18-bar truss geometry [39]

Name	Magnitude (kips)	Direction	Nodes
Case 1	-20	y-axis	1,2,4,6,8

Table 2: 18-bar truss loading

Material Properties	
Density (lbs/in^3)	0.1
Modulus of Elasticity (ksi)	10,000
Constraints	
Allowable tensile stress (ksi)	20
Allowable compressive stress (ksi)	varies
Buckling coefficient (in)	4

Table 3: 18-bar truss design parameters

	[22]	[11]	[31]	[39]	This study: Approach A	This study: Approach B
Total weight (lb)	4,669.72	5,713.00	4,616.82	4,608.04	4,555.90	4,321.52
Relative weight (%)	101.3%	124.0%	100.2%	100%	98.9%	93.8%
Num analyses	?	8	?	176,600	54,632	65,870
Relative analyses (%)	?	0.005%	?	100%	30.9%	37.2%
Max constraint value	?	?	?	1.075	0.999	1.0749
Shape variables (mm)						
x3	22,746	25,263	22,654	23,500	23,380	23,039
y3	3,647	4,122	3,690	5,000	4,632	4,519
x5	15,448	18,984	15,509	16,800	16,410	16,133
y5	2,677	2,614	3,002	3,800	3,612	3,475
x7	9,919	12,266	9,789	10,900	10,490	10,305
y7	1,450	838	1,842	2,500	2,389	2,215
x9	4,597	5,631	4,684	5,300	5,117	5,039
y9	81	434	594	500	769	545
Size variables (in^2)						
A1	11.24	11.34	12.50	12.00	12.5	11.25
A2	15.68	19.28	16.25	18.50	17.75	16.75
A3	7.93	10.97	8.00	5.25	6.00	5.75
A4	6.49	5.30	4.00	4.50	3.75	4.25

Table 4: 18-bar truss results (Case 1)

	[39]	This study: Approach A	This study: Approach B
Total weight (lb)	3,632.94	3,628.81	3,606.60
Relative weight (%)	100.0%	99.9%	99.3%
Num analyses	943,500	106,684	62,532
Relative analyses (%)	100.0%	11.3%	6.6%
Max constraint value	1.013	0.999	1.009
Shape variables (mm)			
x3	24,100	22,718	24,359
y3	2,900	3,046	2,874
x5	17,700	17,125	17,477
y5	2,300	2,413	2,703
x7	11,400	11,031	11,422
y7	1,800	1,951	2,025
x9	5,500	5,516	5,703
y9	500	576	790

Size variables (in^2)			
A1	2.25	2.25	2.25
A2	2.75	2.50	2.50
A3	5.75	6.25	6.50
A4	9.50	9.75	10.00
A5	11.75	11.75	12.00
A6	11.50	11.00	11.00
A7	12.75	14.50	14.00
A8	16.25	14.75	16.25
A9	18.25	19.00	17.75
A10	18.75	19.00	19.75
A11	3.50	3.25	3.25
A12	6.25	6.50	6.25
A13	7.75	7.50	7.00
A14	8.50	8.25	7.75
A15	2.75	3.00	3.50
A16	3.75	3.50	3.25
A17	2.50	2.25	2.25
A18	5.00	4.75	4.25

Table 5: 18-bar truss results (Case 2)

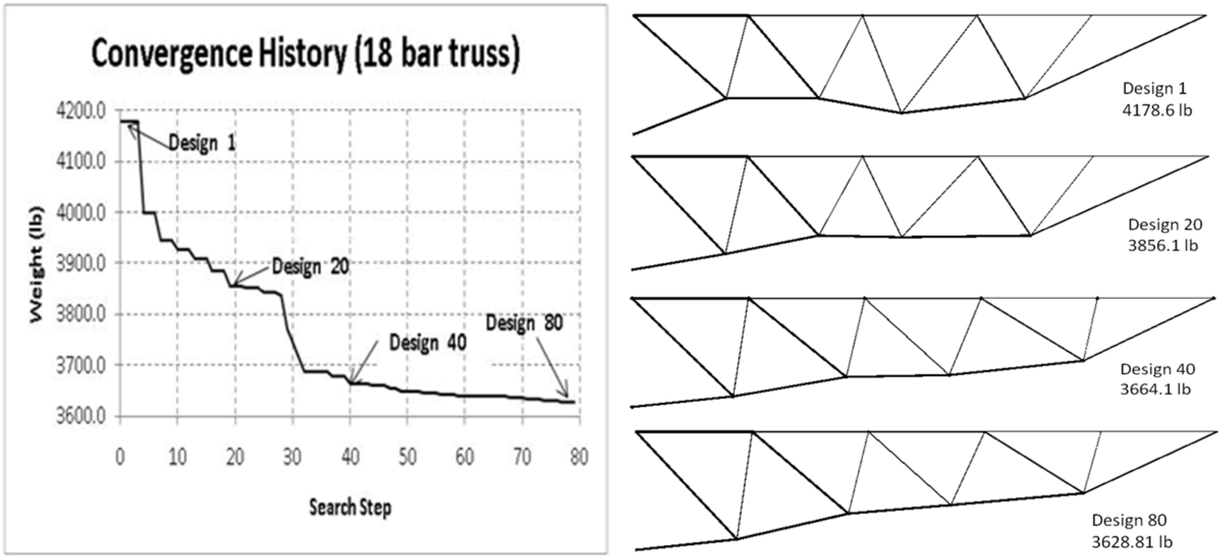

Figure 4: LEFT - Optimization convergence history for the 18-bar truss. Each search step corresponds to the construction / refinement of the surrogate models used in the shape optimization process. RIGHT - Progressive evolution of shape for 18-bar truss by search step.

4.2. Example 2: 77-bar truss

The next example is a single span bridge that crosses an opening of 500 ft (152.4 m) as shown in Figure 5. The bridge consists of 21 panel points (including end supports) equally spaced at 25 ft (7,620 mm). The bridge loading consists of live loads from traffic combined with dead loading from the deck and floor system. This loading is applied as point loads of 60 kips acting on the bottom chord of the truss (Table 6) as a single load case. The truss has 77 members, which are grouped into 39 design variables based on symmetry (Figure 5). A total of 83 discrete sections between W10x12 and W14x730 in the W-shape profile list [48] are the candidate values for the design variables. To enable comparison with prior results, a single continuous shape variable (y_1) is considered, which defines the y-coordinate values for the nodes on the top chord of the truss. The stress and slenderness constraints are based on standard AISC-ASD checks [49]. Furthermore, the maximum displacement of the panel points in any direction is restricted to 10.0 in. (254 mm), which is equal to 1/600 of the total span length (Table 7).

The quality of the BiOPT solution produced is superior to the results reported by Hancebi [49] by approximately 1.4%. It is also worth noting that the proposed method is more than an order of magnitude more efficient in reaching this solution. The convergence history for the BiOPT method is shown in Figure 6.

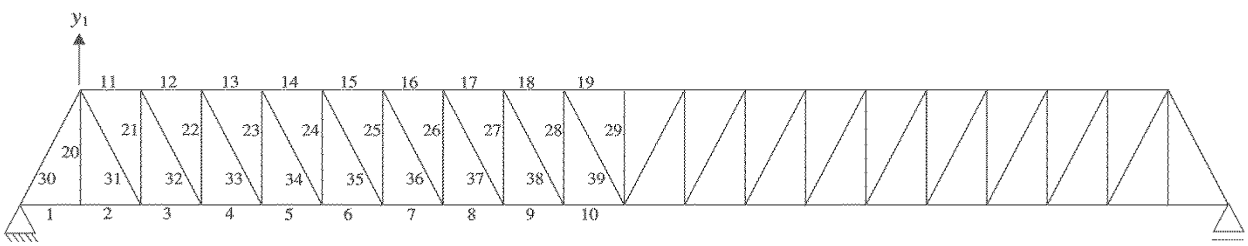

Figure 5: 77-bar truss geometry [50]

Name	Magnitude (kips)	Direction	Nodes
Case 1	-60	y-axis	1-11

Table 6: 77-bar truss loading

Material Properties	
Density (lbs/in^3)	0.1
Modulus of Elasticity (ksi)	29,000
Yield stress (ksi)	36
Constraints	
Allowable tensile stress (ksi)	21.6
Allowable compressive stress (ksi)	varies
Allowable displacement (in)	10

Table 7: 77-bar truss design parameters

	[33]	This study
Total weight (lb)	518,054	511,037
Relative weight (%)	100.0%	98.6%
Num analyses	100,000	7,553
Relative analyses (%)	100%	0.76%
Max constraint value	1.11	0.99
Shape variables (in)		
y1	607.80	608.49
Size variables, ready sections (in^2)		
A1	W10X60 (17.6)	W10X68 (20.0)
A2	W10X60 (17.6)	W10X68 (20.0)
A3	W12X120 (35.3)	W14X132 (38.8)
A4	W12X170 (50.0)	W14X193 (56.8)
A5	W12X210 (61.8)	W14X257 (75.6)
A6	W14X257 (75.6)	W14X283 (83.3)
A7	W14X283 (89.5)	W14X342 (101.0)
A8	W14X311 (91.4)	W14X370 (109.0)
A9	W14X311 (98.7)	W14X398 (117.0)
A10	W14X342 (108.9)	W14X426 (125.0)
A11	W12X120 (35.3)	W14X145 (42.7)
A12	W12X210 (61.8)	W12X210 (61.8)
A13	W14X257 (75.6)	W14X257 (75.6)
A14	W14X283 (89.5)	W12X305 (89.6)
A15	W14X342 (101.0)	W14X342 (101)
Size variables, ready sections (in^2) – cont.		
A16	W14X398 (117.0)	W14X370 (109.0)
A17	W14X398 (124.9)	W14X398 (117.0)
A18	W14X398 (117.0)	W14X426 (125.0)
A19	W14X455 (146.9)	W14X426 (125)
A20	W10X49 (14.4)	W12X50 (14.6)
A21	W14X233 (68.5)	W12X252 (74.0)
A22	W14X211 (62.0)	W12X230 (67.7)
A23	W12X170 (51.7)	W12X190 (55.8)
A24	W12X136 (42.6)	W12X152 (44.7)
A25	W12X120 (38.7)	W12X136 (39.9)
A26	W14X99 (29.1)	W10X100 (29.4)
A27	W14X82 (25.5)	W10X88 (25.9)
A30	W10X68 (21.0)	W14X82 (24.0)
A31	W10X49 (14.4)	W12X50 (14.6)
A32	W14X342 (108.9)	W14X398 (117.0)
A33	W12X120 (38.7)	W12X152 (44.7)
A34	W12X120 (35.3)	W12X136 (39.9)
A35	W14X109 (32.8)	W14X120 (35.3)
A36	W14X109 (32.8)	W14X99 (29.1)
A37	W14X90 (26.5)	W12X79 (23.2)
A38	W12X87 (25.8)	W14X61 (17.9)
A39	W14X74 (21.8)	W12X50 (14.6)

Table 8: 77-bar truss results

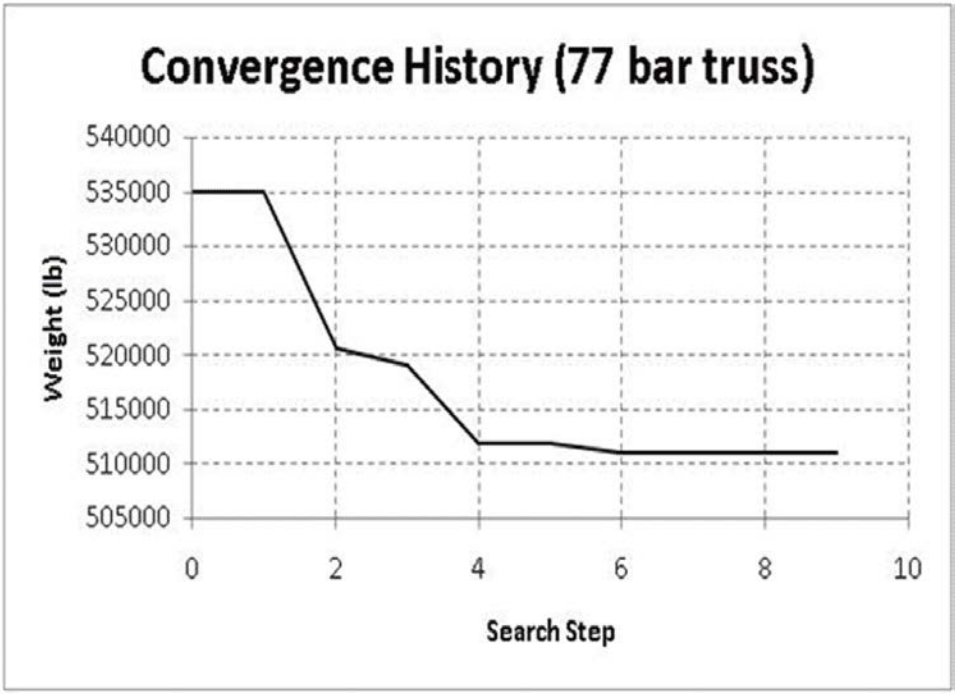

Figure 6: Optimization convergence history for 77-bar truss. Each search step corresponds to the construction / refinement of the surrogate models used in the shape optimization process.

5. Case Study: Arena Roof Trusses

5.1. Background

The main roof trusses for a 30,000 seat arena were selected to test the scalability of the BiOPT method and to compare the method's performance to conventional industry practice. The arena has a sliding roof that fulfills UEFA and FIFA regulations for international football. The main roof trusses span approximately 180 meters and are simply supported as shown in Figure 7. The truss has a variable cross section consisting primarily of circular hollow sections with I sections used for the top chord of the truss. The mid-span deflection of the truss is controlled by pre-tensioning the cables attached to the cantilevered ends of the truss. Two design methods were applied in parallel to optimize shape and sizing of the trusses: (i) the conventional design method of a leading engineering firm and (ii) the BiOPT method described in Section 3. The implementation of each method is explained below and the results are compared in terms of solution quality and process efficiency.

5.2. Problem specification

The objective of the optimization process was to minimize the total weight of the roof trusses while satisfying structural performance criteria for strength and serviceability as summarized in Table 9. The design variables included one shape variable and 17 sizing variables. The shape variable was the mid-span depth of the roof (d) as shown in Figure 7. The sizing variables were created by aggregating the 181 members in each roof truss into groups. All constituent members within a given group possessed the same cross section. This was done for two reasons: (1) to ensure symmetry, continuity, and proportion for the structural elements since the roof structure would be exposed, and (2) to standardize member connections to simplify the fabrication and erection process. The candidate section sizes within each group were chosen from the British Standards Institution catalogue [42].

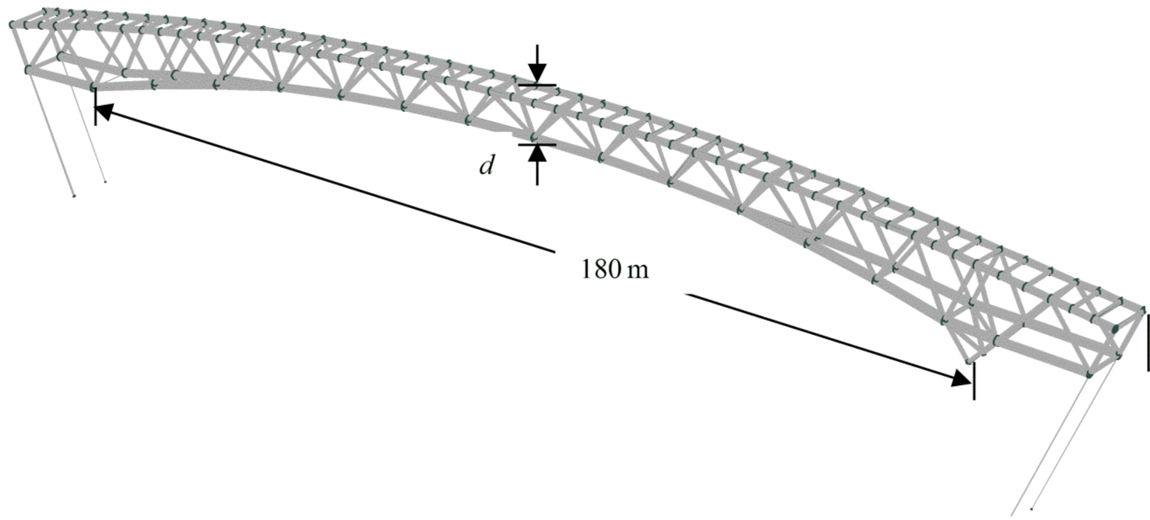

Figure 7: Structural analysis model of the roof truss selected for the shape and sizing optimization study.

Objective	*Minimize* steel weight
Shape Variable	• Mid-span depth of truss (*d*) • Range: 3m – 15m
Sizing Variables	• 181 members aggregated into 17 variables • 10-30 candidate sections per variable
Constraints	• Member strength [51] • Member deflection (span / 360 for SLS load cases) • Global deflection limit (500 mm at mid-span)

Table 9: Overview of shape and sizing optimization problem formulation for the arena roof trusses.

5.3. Conventional Process

The steps involved in the conventional member sizing optimization process were similar to those outlined in Section 1. First, a detailed finite element analysis model of the structure was created. It included 38 unique loading combinations, consisting of the weight of the moving roof, cladding, equipment, and the structural members, as well as live loads from wind and snow. The initial configuration of member sizes was determined based on the best judgment of the engineering team from their past experience with similar stadium roof structures. After completing the Finite Element Analysis (FEA) of the structure, the engineering team checked the structural responses against the design requirements for safety and serviceability as specified in Table 9. Finally, the engineering team reviewed the results and modified the shape of the structure and/or the sizes of constituent members. This process was repeated until the team was satisfied with the design configuration.

5.4. BiOPT Implementation

In parallel with the conventional design process, we optimized the shape and sizing configuration of the roof truss using the BiOPT method described in Section 3. The method was implemented in ModelCenter® [52], a commercial software package. It allows users to bring commercial or proprietary software tools into a common environment using software "wrappers" or "plug-ins". The six components created in ModelCenter to implement the process are described below.

Step 1 - Geometry: Generates the node coordinates for a 1-D centerline representation of all the truss members based on the shape variable values specified. The geometry for the truss was created and controlled using the parametric computer-aided design (CAD) software Digital Project [53].

Step 2 - FEA: (i) Updates the FEA model geometry in GSA [54] based on the input from the Geometry component; (ii) executes the FEA and stores the desired structural responses (e.g., deflections, member forces and moments). See Section 3.1, step 2 of the FCD method.

Step 3 - ASDCheck: Calculates the strength utilization ratio for each member based on the British engineering code of practice [55]. The input to this component is each member's length, section size, and the responses (e.g., forces). A utilization ratio of less than unity indicates that the strength of the member is adequate for the defined loading. See Section 3.1, step 2 of the FCD method.

Step 4 - PreProcessor: (i) Scales each constraint type (e.g., strength utilization, member deflection, global deflection) to unity based on the allowable value; (ii) determines if the global displacement constraint is satisfied; (iii) calculates the critical constraint for each sizing design variable. See Section 3.1, steps 3-6 of the FCD method.

Step 5 - SizingOPT: (i) Modifies the member sizing variables based on the corresponding critical constraint values; (ii) checks whether the design configuration is unique and enters 'oscillation mode' if necessary; (iii) concludes the optimization process if the convergence criteria have been met. See Section 3.1, steps 7-10 of the FCD method.

Step 6 - ShapeOPT: (a) Modifies the shape variables based on the results of the SizingOPT process to find the optimal (e.g., minimum weight) shape of the structure for a given topology. See Section 3.2, steps 1-9 of the SEQOPT method.

5.5. Results

The performance of the BiOPT method on the project was compared to the conventional design process in terms of process efficiency and solution quality. In terms of process efficiency, the BiOPT method took 30 man-hours more than the traditional design method because the software 'wrappers' necessary to integrate the CAD and FEA software into the ModelCenter environment had to be developed. These components were designed for general use. The same ModelCenter implementation was used to complete the case study and all the numerical examples without requiring additional software development; the only modification required was selecting the appropriate CAD and FEA models. Therefore, the time required to set up BiOPT for subsequent industry applications would likely be comparable to the conventional process.

Once set up, the engineering team evaluated a total of 36 sizing and 8 shape configurations using the conventional process, requiring a total of 248 man hours. In contrast, the BiOPT method evaluated 15,376 design alternatives in approximately five percent of the time required by conventional practice. The efficiency of the BiOPT process was facilitated by parallelizing the calculations over a network of 16 computing nodes. Each node was equipped with two Quad Core Xeon E5440 processors (2.83 GHz) and 16 GB of memory. See Table 10 for a summary of the process efficiency of each method.

In terms of solution quality, the best design found by the BiOPT method had a total steel weight of 642 metric tons and satisfied all the design constraints. This represented a weight reduction of 24% compared to the best design that was achieved following conventional practices. This weight reduction translated into an estimated cost savings of approximately $1.2 million USD for the steelwork ($600,000 USD per roof structure) assuming a unit cost of structural steel of $6,000 USD per metric ton.

DESIGN METHOD	SIZING DESIGN CYCLE		SHAPE DESIGN CYCLE		TOTAL DURATION (HRS)
	Avg. cycle duration	Number of Cycles	Avg. cycle duration	Number of Cycles	
Conventional Practice	4 hrs	36	13 hrs	8	248 hrs
BiOPT Method	3 sec	15,376	12 min	62	13 hrs

Table 10: Comparison of process efficiency for arena roof trusses case study

6. Summary and Conclusions

This paper presents the BiOPT method for shape and sizing optimization of truss and frame structures. The proposed method decomposes the problem into two hierarchical levels: shape and sizing. A specialized deterministic algorithm known as Fully Constrained Design (FCD) operates on the discrete sizing variables. FCD is nested within SEQOPT, a gradient-based optimization method that operates on the continuous shape variables. BiOPT's unique combination of algorithms was tested against existing methods, using a set of numerical examples demonstrating the applicability of the method to determinate structures and an industry case study project showing the use of the method for an indeterminate structure. The main conclusions of the study are:

- The proposed method is capable of handling both continuous shape variables and discrete sizing variables.
- The sizing algorithm (FCD) does not require the first derivative of the objective and constraint functions with respect to the design variables. It is, therefore, readily applicable to problems where the objective and constraint functions are discontinuous or not easily expressed in terms of the design variables.

- For the truss examples, the quality of the solutions produced by BiOPT is superior to those produced by existing optimization techniques based on the benchmarking studies conducted (2.8% less steel weight on average).
- The computational efficiency of BiOPT is superior to existing techniques, except for Hansen and Vanderplaat's gradient-based approach with force approximations, which produced solutions of inferior quality compared to the other approaches listed.
- BiOPT compares favorably to the conventional design process of a leading engineering firm based on a parallel case study involving the design of two large roof trusses for a 30,000 seat arena. The BiOPT solution resulted in an estimated cost savings of $1.2 million USD compared to the conventional process and required significantly less time to complete.

The significant savings achieved in the case study project demonstrate the potential of the BiOPT method to improve design process efficiency and the quality of the resulting product. Further industry applications will be required to comment more generally on the performance and robustness of the proposed method compared to manual design iteration methods commonly used in industry today. The benchmarking examples in this paper included determinate structures with a limited number of shape variables (8 and 1, respectively). Additional benchmarking studies are planned to compare the performance of BiOPT to other formal methods when applied to determinate structures involving a larger number of shape variables as well as indeterminate structures.

Further work is planned to improve the practicality of the method by enabling BiOPT to consider more accurate cost objective functions that explicitly consider procurement, fabrication, and erection processes. Finally, to make the method more useful for conceptual design, the authors plan to explore the possibility of extending the proposed method to address topology as well as shape and sizing variables.

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List of Figures

Figure 1: BiOPT architecture and data flow. Coupling above the diagonal represents sequential execution while coupling below the diagonal represents iteration.

Figure 2: LEFT - Flowchart of the FCD algorithm (sizing optimization).

Figure 3: 18-bar truss geometry [39]

Figure 4: LEFT - Optimization convergence history for the 18-bar truss. Each search step corresponds to the construction / refinement of the surrogate models used in the shape optimization process. RIGHT - Progressive evolution of shape for 18-bar truss by search step.

Figure 5: 77-bar truss geometry [50]

Figure 6: Optimization convergence history for 77-bar truss. Each search step corresponds to the construction / refinement of the surrogate models used in the shape optimization process.

Figure 7: Structural analysis model of the roof truss selected for the shape and sizing optimization study.

List of Tables

Table 1: Initial sizing configuration used by BiOPT for the numerical examples.

Table 2: 18-bar truss loading

Table 3: 18-bar truss design parameters

Table 4: 18-bar truss results (Case 1)

Table 5: 18-bar truss results (Case 2)

Table 6: 77-bar truss loading

Table 7: 77-bar truss design parameters

Table 8: 77-bar truss results

Table 9: Overview of shape and sizing optimization problem formulation for the arena roof trusses.

Table 10: Comparison of process efficiency for arena roof trusses case study